SWAMI VIVEKANANDA SCHOOL OF

ENGINEERING & TECHNOLOGY

SUBJECT NOTE – STRENGTH OF MATERIAL

SEMESTER – 3RD

LECTURER NAME – ER. KUNAL PRADHAN

STRENGTH OF MATERIALS # MOM (Mechanics of Materials)

(3) (1) Simple stresses and strains :

(1) Study of strength of materials :

The strength of materials deals with the ability of various types of materials to resist its failure and there behavior under the action of forces

(2) Mechanical Properties of materials :

Elasticity : It is the property of a material to regain its original state after deformation when the external forces are removed. This property is desirable for materials used in tools and machines. It may be noted that steel is more elastic than rubber

Plasticity : It is property of a material which retains the deformation produced under load permanently. This property of material is necessary for forgings, in stamping images on coins, and in ornamental work

Compressibility : It is that property of a material by virtue of which materials undergo a material change in volume with the change in pressure

Hardness : It is a very important property of the metals. It embraces many different properties such as resistance to wear, scratching, deformation and machinability etc. It also means the ability of a metal to cut another metal

Toughness : It is the property of a material to resist fracture due to high impact loads like hammer blows. The toughness of a material decreases when it is heated. This property is desirable in parts subjected to shock and impact loads

Stiffness: It is the ability of a material to resist deformation under stress. The modulus of elasticity is the measure of stiffness

Brittleness: It is the property of a material opposite to ductility. It is the property of breaking of a material with little permanent distortion. cast iron is a brittle material

Ductility: It is property of a material enabling it to be drawn into wire with the application of a tensile force. Examples are mild steel, copper, aluminium, nickel, zinc, tin and lead

Malleability: It is a special case of ductility which permits materials to be rolled or hammered into thin sheets. Examples are, lead, soft steel, wrought iron, copper and aluminium,

Creep: When a part is subjected to a constant stress at high temperature for a long period of time, it will undergo a slow and permanent deformation called creep. This property is considered in ~~designing~~ designing internal combustion engines, boilers and turbines

Fatigue: When a material is subjected to repeated stresses, it fails at stresses below the yield point stresses. Such type of material is known as fatigue. The property is considered in designing shafts, connecting rods, spring, gears etc.

(1) **Stress:** When some external system of forces or loads act on a body, the internal forces (equal and opposite) are set up at various sections of the body, which resist the external forces. This internal force per unit area at any section of the body is known as stress. Mathematically,

Stress, $\sigma = \frac{P}{A}$ where P = force or load acting on the body and, A = cross sectional area of the body

(2) **Strain:** When a system of forces act on a body, it undergoes some deformation. This deformation per unit length is known as strain. Mathematically,

strain, $\varepsilon = \delta l / l$

where δl = change in length of the body, and
l = original length of the body

(3) **Types of stresses and strains:**

* **Tensile stress and strain:** When a body is subjected to two equal and opposite axial pulls, as a result of which the body tends to extend its length, the stress and strain induced is known as tensile stress and tensile strain.

Compression stress and strain :

When a body is subjected to two equal and opposite axial pushes as a result of which the body tends to decrease its length, the stress and strain induced is known as compressive stress and compressive strain.

Shear stress and strain :

When a body is subjected to two equal and opposite forces acting tangentially across the resisting section as a result of which the body tends to shear off the section then the stress and strain induced is called shear stress and strain.

(i) Longitudinal or Primary or linear strain :

The deformation of the body per unit length in the direction of the force (P/A) is known as longitudinal or primary or linear strain.

(ii) Secondary or lateral strain :

Every direct stress is always accompanied by a strain in its own direction and an opposite kind of strain in every direction, at right angles to it. Such a strain is known as secondary or lateral strain.

(iii) Poisson's Ratio :

The ratio of lateral strain to linear strain is called Poisson's Ratio

③ Volumetric strain :

The ratio of change in volume to the original volume is known as volumetric strain.

The volumetric strain of a rectangular bar of length (l), breadth (b) and thickness (t) and subjected to an axial force (P) is given by,

$$\frac{\delta V}{V} = \frac{P}{btE}\left(1-\frac{2}{m}\right) = \epsilon_l\left(1-\frac{2}{m}\right) = \epsilon(1-2\mu)$$

$\left[\mu = \frac{1}{m}\right]$

④ Change in dimensions of a thin cylindrical shell :

When a thin cylindrical shell is subjected to an internal pressure, its walls will subjected to lateral strain, the effect of which is to cause some change in the dimensions (length and diameter) of the shell. The circumferential strain is given by

$$\epsilon_c = \frac{pd}{2tE}\left(1-\frac{1}{2m}\right) = \frac{pd}{4tE}(2-\mu)$$

and longitudinal strain,

$$\epsilon_l = \frac{pd}{2tE}\left(\frac{1}{2}-\frac{1}{m}\right) = \frac{pd}{4tE}(1-2\mu)$$

where E = Young's modulus for the shell material,

and $1/m$ = Poisson's Ratio

- The change in diameter is given by,

$$\delta d = \epsilon_c d = \frac{pd^2}{2tE}\left(1-\frac{1}{2m}\right) = \frac{pd^2}{4tE}(2-\mu)$$

and change in length

$$\delta l = \epsilon_l l = \frac{pdl}{2tE}\left(\frac{1}{2}-\frac{1}{m}\right) = \frac{pdl}{4tE}(1-2\mu)$$

(4) Hooke's law :

It states "when a material is loaded within its elastic limit, the stress is proportional to the strain" mathematically,

$$\frac{stress}{strain} = E = constant$$

(5) Elastic constants :

Young modulus or modulus of Elasticity :

Hooke's law states that when a material is loaded within elastic limit, the stress is directly proportional to strain.

$\sigma \propto \varepsilon$ or $\sigma = E\varepsilon$ or, $E = \frac{\sigma}{\varepsilon} = \frac{P \times l}{A \times \delta l}$

where E is known as Young's modulus or modulus of elasticity

Shear modulus or modulus of Rigidity :

It has been found experimentally that within elastic limit the shear stress is directly proportional to shear strain. mathematically,

$\tau \propto \phi$ or, $\tau = C\phi$ or $\tau / \phi = C$

where, τ = shear stress, ϕ = shear strain and C = constant of proportionality known as shear modulus or modulus of rigidity.

Bulk modulus :

When a body is subjected to three mutually perpendicular stresses, of equal intensity, the ratio of direct stress to the corresponding volumetric strain is known as bulk modulus. It is usually denoted by K.

① Derivation for the Relationship between elastic constants :

[When a cube is subjected to three mutually perpendicular tensile stresses of equal intensity (σ), then the volumetric strain,

$$\frac{\delta V}{V} = \frac{3\sigma}{E}\left(1 - \frac{2}{m}\right) = \frac{3\sigma}{E}(1-2\mu)$$]

The Relation between bulk modulus and Young's modulus is given by,

$$K = \frac{mE}{3(m-2)} = \frac{E}{3(1-2\mu)}$$

The Relation between modulus of elasticity (E) and modulus of rigidity (C) is given by,

$$C = \frac{mE}{2(m+1)} = \frac{E}{2(1+\mu)}$$

And the Relation between E, C and K is given by,

$$E = \frac{9KC}{3K+C}$$

PROBLEM :

(1) A hollow cylinder 2 m long has an outside diameter of 50 mm and inside diameter of 30 mm. If the cylinder is carrying a load of 25 kN, find the stress in the cylinder. Also find the deformation of the cylinder if the value of modulus of elasticity for the cylinder material is 100 GPa.

Solution : Given : Length (l) = 2 m = 2×10^3 mm

Outside diameter (D) = 50 mm, Inside diameter (d) = 30 mm,

Load (P) = 25 kN = 25×10^3 N and modulus of elasticity (E) = 100 GPa = 100×10^3 N/mm²

We know that cross sectional area of the hollow cylinder,

$$A = \frac{\pi}{4} \times (D^2 - d^2) = \frac{\pi}{4}\left[(50)^2 - (30)^2\right]$$

$$= 1257 \text{ mm}^2$$

and stress in the cylinder,

$$\sigma = \frac{P}{A} = \frac{25 \times 10^3}{1257} = 19.9 \text{ N/mm}^2 = 19.9 \text{ MPa} \quad \underline{\text{Ans}}$$

And deformation of the cylinder,

$$\delta l = \frac{P\,l}{A\,E} = \frac{(25 \times 10^3) \times (2 \times 10^3)}{1257 \times (100 \times 10^3)} = 0.4 \text{ mm} \quad \underline{\text{Ans}}$$

(02) A load of 5 kN is to be raised with the help of a steel wire. Find the minimum diameter of the steel wire, if the stress is not to exceed 100 MPa.

<u>Solution</u>: Given: Load (P) = 5 kN = 5×10^3 N and stress (σ) = 100 MPa = 100 N/mm²

Let d = Diameter of the wire in mm.

We know that stress in the steel wire (σ),

$$100 = \frac{P}{A} = \frac{5 \times 10^3}{\frac{\pi}{4} \times (d)^2} = \frac{6.366 \times 10^3}{d^2}$$

$$d^2 = \frac{6.366 \times 10^3}{100} = 63.66$$

$$d \cong 7.98 \cong 8 \text{ mm} \quad \underline{\text{Ans}}$$

(03) In an experiment, a steel specimen of 13 mm diameter was found to elongate 0.2 mm in a 200 mm gauge length when it was subjected to a tensile force of 26.8 kN. If the specimen was tested within the elastic range, what is the value of Young's modulus for the steel specimen?

Solution: Given: Diameter $(d) = 13$ mm, Elongation $(\delta l) = 0.2$ mm, Length $(l) = 200$ mm and Force $(P) = 26.8$ kN

Let E = value of Young's modulus for the steel specimen.

We know that cross sectional area of the specimen

$$A = \frac{\pi}{4} \times (d)^2 = \frac{\pi}{4} \times (13)^2 = 132.73 \text{ mm}^2$$

and elongation of the specimen (δl),

$$0.2 = \frac{P \cdot l}{A \cdot E} = \frac{26.8 \times 200}{132.73 \times E} = \frac{40.38}{E}$$

$$\text{or, } E = \frac{40.38}{0.2} = 201.9 \text{ kN/mm}^2 = 201.9 \text{ GPa} \quad \text{Ans.}$$

(04) A hollow steel tube 2.5 m long has external diameter of 120 mm. In order to determine the internal diameter, the tube was subjected to a tensile load of 400 kN and extension was measured to be 2 mm. If modulus of elasticity for the tube material is 200 GPa, determine the internal diameter of the tube.

Solution: Given, Length $(l) = 2.5$ m $= 2.5 \times 10^3$ mm, Load $(P) = 400$ kN $= 400 \times 10^3$ N, External diameter $(D) = 120$ mm, Extension $(\delta l) = 2$ mm and modulus of elasticity $E = 200$ GPa

$$\text{or, } E = 200 \times 10^3 \text{ N/mm}^2$$

Let, d = Internal diameter of the tube in mm.

We know that area of the tube,

$$A = \frac{\pi}{4}\left[(120)^2 - d^2\right] = 0.7854\left[(120)^2 - d^2\right]$$

And extension of the tube (δl),

$$2 = \frac{P \cdot l}{A \cdot E} = \frac{(400 \times 10^3) \times (2.5 \times 10^3)}{0.7854\left[(120)^2 - d^2\right](200 \times 10^3)}$$

$$= \frac{8913}{14400 - d^2}$$

$$\text{or, } 28800 - 2d^2 = 8913 \quad \text{or, } 2d^2 = 28800 - 8913 = 19887$$

$$\text{or, } d^2 = \frac{19887}{2} = 9943.5 \quad \text{or, } d = 99.7 \text{ mm} \quad \text{Ans.}$$

(25) Two wires, one of steel and the other of copper, are of the same length and are subjected to the same tension. If the diameter of the copper wire is 2 mm, find the diameter of the steel wire, if they are elongated by the same amount. Take E for steel as 200 GPa and that for copper as 100 GPa.

<u>Solution:</u> Given: diameter of copper wire $(d_c) = 2$ mm,

modulus of elasticity for steel $(E_s) = 200$ GPa $= 200 \times 10^3$ N/mm^2

and " " copper $(E_c) = 100$ GPa $= 100 \times 10^3$ N/mm^2

Let, d_s = diameter of the steel wire,

l = length of both the wires and

P = Tension applied on both the wires.

Area of the copper wire, $A_c = \frac{\pi}{4} \times (d_c)^2$

$= \frac{\pi}{4} \times (2)^2 = 3.142$ mm^2

and area of steel wire $A_s = \frac{\pi}{4} \times (d_s)^2 = 0.7854\, d_s^2$

Increase in the length of copper wire,

$$\delta l_c = \frac{Pl}{E_c A_c} = \frac{Pl}{3.142 \times (100 \times 10^3)} = \frac{Pl}{314.2 \times 10^3}$$

and increase in the length of the steel wire,

$$\delta l_s = \frac{Pl}{A_s E_s} = \frac{Pl}{0.7854\, d_s^2 \times (200 \times 10^3)} = \frac{Pl}{157.1 \times 10^3 \times d_s^2}$$

Since both the wires are elongated by the same amount,

$$\frac{Pl}{314.2 \times 10^3} = \frac{Pl}{157.1 \times 10^3 \times d_s^2}$$

or, $d_s^2 = \frac{314.2}{157.1} = 2$

$\therefore d_s = \sqrt{2} = 1.41$ mm. <u>Ans:</u>

(06) A steel bar 2 m long, 40 mm wide and 20 mm thick is subjected to an axial pull of 160 kN in the direction of its length. Find the changes in length, width and thickness of the bar. Take E = 200 GPa and Poisson's ratio = 0.3

Solution: Given: Length $(l) = 2\,m = 2\times10^3\,mm$, Width $(b) = 40\,mm$, Thickness $(t) = 20\,mm$, Axial pull $(P) = 160\,kN = 160\times10^3\,N$, Modulus of Elasticity $(E) = 200\,GPa = 200\times10^3\,N/mm^2$ and Poisson ratio $(\frac{1}{m}) = \mu = 0.3$

change in length,

$$\delta l = \frac{Pl}{AE} = \frac{(160\times10^3)\times(2\times10^3)}{(40\times20)\times(200\times10^3)} = 2\,mm \quad \text{Ans:}$$

change in width,

Linear strain, $\varepsilon = \frac{\delta l}{l} = \frac{2}{2\times10^3} = 0.001$

and Lateral strain $= \frac{1}{m}\times\varepsilon = 0.3\times0.001 = 0.0003$

$\therefore$ change in width, $\delta b = b\times$ Lateral strain

$= 40\times0.0003 = 0.012\,mm$ Ans:

change in thickness,

$\delta t = t\times$ Lateral strain

$= 20\times0.0003 = 0.006\,mm$ Ans:

(07) A metal bar 50 mm × 50 mm in section is subjected to an axial compressive load of 500 kN. If the contraction of a 200 mm gauge length was found to be 0.5 mm and the increase in thickness 0.04 mm, find the values of Young's modulus and Poisson's ratio for the bar material

Solution: Given: Width $(b) = 50\,mm$, Thickness $(t) = 50\,mm$, Axial compressive load $(P) = 500\,kN = 500\times10^3\,N$, Length $(l) = 200\,mm$, change in length $(\delta l) = 0.5\,mm$ and change in thickness $(\delta t) = 0.04\,mm$

Let, E = Value of Young's modulus for the bar material

Contraction of the bar (δl),

$$0.5 = \frac{Pl}{AE} = \frac{(500\times10^3)\times200}{(50\times50)\times E} = \frac{40\times10^3}{E}$$

$$\therefore E = \frac{40\times10^3}{0.5} = 80\times10^3 \text{ N/mm}^2 = 80 \text{ GPa} \quad \underline{\text{Ans:}}$$

Linear strain, $\varepsilon = \frac{\delta l}{l} = \frac{0.5}{200} = 0.0025$

Let, Poisson's ratio $= \frac{1}{m} = \mu$

Lateral strain $= \frac{1}{m} \times$ linear strain

$$= \frac{1}{m} \times 0.0025$$

Increase in thickness (δt),

$$0.04 = t \times \text{Lateral strain} = 50 \times \frac{1}{m} \times 0.0025$$

$$= \frac{0.125}{m}$$

$$\text{or, } \frac{1}{m} = \frac{0.04}{0.125} = 0.32 \quad \underline{\text{Ans:}}$$

(08) A steel bar 2 m long, 20 mm wide and 15 mm thick is subjected to a tensile load of 30 kN. Find the increase in volume, if Poisson ratio is 0.25 and Young's modulus is 200 GPa.

Solution: Given: Length (l) = 2 m = 2×10^3 mm,

Width (b) = 20 mm, Thickness (t) = 15 mm,

Tensile load (P) = 30 kN = 30×10^3 N,

Poisson's ratio ($\frac{1}{m}$) = 0.25 or $m = 4$ and Young's modulus of elasticity (E) = 200 GPa = 200×10^3 N/mm²

Let, δV = Increase in volume of the bar

Original volume of the bar,

$V = l.b.t$ = [illegible] mm³

and $\frac{\delta V}{V} = \frac{P}{b.t.E}\left(1 - \frac{2}{m}\right)$ [illegible]

[illegible]

[illegible]

[illegible] A metal bar [illegible] is subjected to an axial pull in the direction of its length. [illegible] the increase in volume of the bar is [illegible] mm³. Find the magnitude of the pull. Take [illegible] and E = [illegible]

Solution: Given: Length (l) = [illegible] mm, width (b) = [illegible] mm, Thickness (t) = [illegible] mm, Increase in volume [illegible] mm³, [illegible] and modulus of elasticity (E) = [illegible]

Let P = Magnitude of the pull in kN.

Original volume of the metal bar,

V = l.b.t = [illegible] mm³

and $\frac{\delta V}{V} = \frac{P}{b.t.E}\left(1 - \frac{2}{m}\right)$ [illegible]

[illegible]

[illegible]

[illegible] A steel bar [illegible] is subjected to an axial pull of [illegible] Find the change in length, width and volume of the bar [illegible] Poisson's ratio [illegible]

Solution: Given: Width (b) = [illegible] mm, Thickness (t) = [illegible] mm, Length (l) = [illegible] mm, Axial pull (P) = [illegible], Poisson's ratio (1/m) = [illegible] and modulus of elasticity (E) = [illegible] change in length,

$$\delta l = \frac{Pl}{AE} = \frac{200\times10^3 \times (1.2\times10^3)}{(50\times50)\times(200\times10^3)} = 0.48 \text{ mm} \quad \underline{Ans.}$$

change in width,

Linear strain $\varepsilon = \frac{\delta l}{l} = \frac{0.48}{1.2\times10^3} = 0.0004$

and lateral strain $= \frac{1}{m}\times\varepsilon$

$= 0.3\times0.0004 = 0.00012$

$\therefore$ change in width, $\delta b = b\times$ lateral strain

$= 50\times0.00012 = 0.006$ mm $\underline{Ans.}$

change in volume,

Volume of the bar,

$V = l\cdot b\cdot t = (1.2\times10^3)\times50\times50$

$= 3\times10^6$ mm^3

and $\frac{\delta V}{V} = \frac{P}{btE}\left(1-\frac{2}{m}\right) = \frac{200\times10^3}{50\times50\times(200\times10^3)}\left[1-(2\times0.3)\right]$

$= 0.00016$

$\delta V = 0.00016\times V = 0.00016\times3\times10^6 = 480$ mm^3 $\underline{Ans.}$

(iii) If the values of modulus of elasticity and Poisson's ratio for an alloy body is 150 GPa and 0.25 respectively, determine the values of bulk modulus for the alloy.

Solution: Given: modulus of Elasticity (E) = 150 GPa $= 150\times10^3$ N/mm^2 and Poisson's Ratio $\left(\frac{1}{m}\right) = 0.25$

or, $m = 4$

Bulk modulus for the alloy,

$$K = \frac{mE}{3(m-2)} \text{ or } \frac{E}{3(1-2\mu)}$$

$$= \frac{4\times150\times10^3}{3(4-2)} = 100\times10^3 \text{ N/mm}^2$$

$= 100$ GPa $\underline{Ans.}$

(16) For a given material, Young's modulus is 120 GPa and modulus of rigidity is 40 GPa. Find the bulk modulus and lateral contraction of a round bar of 50 mm diameter and 2.5 m long, when stretched 2.5 mm. Take Poisson's Ratio as 0.25.

Solution: Given: Young's modulus (E) = 120 GPa = 120×10^3 N/mm^2, modulus of rigidity (C) = 40 GPa = 40×10^3 N/mm^2, Diameter (d) = 50 mm, Length (l) = 2.5 m = 2.5×10^3 mm, Linear stretching or change in length (δl) = 2.5 mm and Poisson's Ratio = 0.25 or m = 4

Bulk modulus of the bar,

$$K = \frac{mE}{3(m-2)} = \frac{4 \times (120 \times 10^3)}{3(4-2)} = 80 \times 10^3 \text{ N/mm}^2$$

$= 80$ GPa Ans.

Lateral contraction of the bar,

Let δd = Lateral contraction of the bar (or change in diameter)

Linear strain $\varepsilon = \frac{\delta l}{l} = \frac{2.5}{2.5 \times 10^3} = \frac{1}{1000}$

$= 0.001$

and Lateral strain, $\frac{\delta d}{d} = \frac{1}{m} \times \varepsilon$

$= 0.25 \times 0.001 = 0.25 \times 10^{-3}$

$\therefore \delta d = d \times (0.25 \times 10^{-3}) = 50 \times (0.25 \times 10^{-3})$

$= 0.0125$ mm Ans.

(17) An alloy specimen has a modulus of elasticity of 120 GPa and modulus of rigidity of 45 GPa. Determine the Poisson's Ratio of the material.

Solution: Given: modulus of elasticity (E) = 120 GPa and modulus of rigidity (C) = 45 GPa

Let, $\frac{1}{m}$ = Poisson's Ratio of the material

∴ Modulus of rigidity $C = 45 = \frac{mE}{2(m+1)}$

or, $45 = \frac{m \times 120}{2(m+1)} = \frac{120m}{2m+2}$

or, $90m + 90 = 120m$

or, $30m = 90$

or, $m = 3$

or, $\frac{1}{m} = \mu = \frac{1}{3}$ **Ans.**

Q4. In an experiment, a bar of 30 mm is subjected to a pull of 60 kN. The measured extension on gauge length of 200 mm is 0.09 mm and the change in diameter is 0.0039 mm. Calculate the Poisson's ratio and the values of the three moduli.

Solution: Given: Diameter $(d) = 30$ mm, Pull $(P) = 60$ kN $= 60 \times 10^3$ N, Length $(l) = 200$ mm, Extension $(\delta l) = 0.09$ mm and change in diameter $(\delta d) = 0.0039$ mm

Poisson's ratio,

Linear strain, $\varepsilon = \frac{\delta l}{l} = \frac{0.09}{200} = 0.00045$

and lateral strain, $= \frac{\delta d}{d} = \frac{0.0039}{30} = 0.00013$

$$\frac{1}{m} = \frac{\text{Lateral strain}}{\text{Linear strain}} = \frac{0.00013}{0.00045} = 0.289 \text{ Ans.}$$

Let, E = Value of Young's modulus

Area of the bar,

$$A = \frac{\pi}{4} \times d^2 = \frac{\pi}{4} \times (30)^2 = 706.9 \text{ mm}^2$$

and extension of the bar (δl),

$$0.09 = \frac{P \cdot l}{A \cdot E} = \frac{(60 \times 10^3) \times 200}{706.9 E} = \frac{17 \times 10^3}{E}$$

∴ $E = 17 \times 10^3 / 0.09 = 188.9 \times 10^3$ N/mm²

$= 188.9$ GPa **Ans.**

Poisson's Ratio, $\frac{1}{m} = 0.289$

or, $m = \frac{1}{0.289} = 3.46$

and value of modulus of rigidity,

$$C = \frac{m \cdot E}{2(m+1)} = \frac{3.46 \times (188.7 \times 10^3)}{2(3.46+1)} \ N/mm^2$$

$$= 73.2 \times 10^3 \ N/mm^2 = 73.2 \ GPa \ \text{Ans:}$$

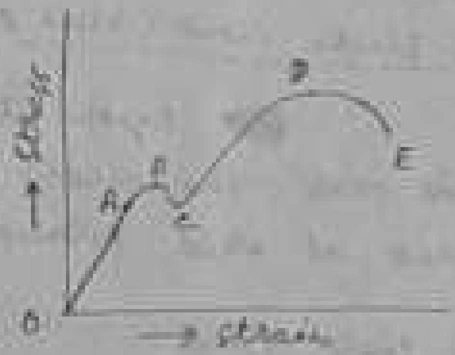
Value of bulk modulus:

$$K = \frac{m \cdot E}{3(m-2)} = \frac{3.46 \times (188.7 \times 10^3)}{3(3.46-2)} \ N/mm^2$$

$$= 149.1 \times 10^3 \ N/mm^2 = 149.1 \ GPa \ \text{Ans:}$$

(1) Application of stress and strain in engineering field:

(2) Stress strain diagram for a mild steel under Tensile test:



The stress strain diagram for a mild steel specimen under tensile test is shown in figure. We see that from O to A is a straight line which represents that the stress is proportional to the strain. It is thus obeying that Hooke's law holds good upto point A, which is called elastic limit. When the material is stress beyond this limit (Point A), then the strain increases more quickly than the stress. The points B and C are called upper yield point and lower yield point respectively. The stress corresponding to point D is called the ultimate stress. After the specimen has reached the ultimate stress, a neck is formed which decreases the cross sectional area of the specimen

A little consideration will show that the stress (or load) necessary to break away the specimen at Point E is less than the ultimate stress. The stress corresponding to Point E is called breaking stress.

(1) Elastic limit:

For a given section there is a limiting value of force up to and within which the deformation entirely disappears on the removal of force. The value of intensity of stress corresponding to this limiting force is called elastic limit of the material.

(2) Limit of Proportionality:

Up to this amount of stress, stress is proportional to strain (Hooke's law), so the stress-strain graph is a straight line and the gradient will be equal to the elastic modulus of the material.

(3) Elastic limit (Yield strength):

Beyond this limit, permanent deformation will occur. The elastic limit is therefore the lowest stress at which permanent deformation can be measured.

(4) Yield stress:

The value of stress at yield point or at the yield strength. A yield point is the material property defined as the stress at which a material begins to deform plastically.

(5) Ultimate stress:

It is the ratio of ultimate load and original cross sectional area. Above the ultimate stress a neck is formed which decreases the cross sectional area of the specimen.

The maximum of the stress tensile, compressive, or shearing stress that a given unit area of a certain material is expected to bear without failing also called ultimate stress.

⑪ Breaking stress :

The stress required to fracture a material whether by tension, compression or shear

$$\text{Breaking stress} = \frac{\text{Breaking load}}{\text{original cross sectional area}}$$

⑫ Designation of yield stress :

When a specimen is loaded beyond the elastic limit the stress increases and reach a point at which the material starts yielding this stress is called yield stress

⑬ Working stress :

Working stress = Yield stress / Factor of safety

Introduction to stress-strain curve :

⑭ Behaviour of ductile materials under direct loads :

In case of ductile materials stress-strain curve follows exactly same path as in tensile test upto and even slightly beyond yield point. For larger values the curves diverge there will not be necking in case of compression test

For MS, mild steel

⑮ Behaviour of brittle materials :

For most brittle materials ultimate compression stress in compression is much larger than in tension. It is because of flaws and cracks present in-

brittle materials which weaken the material in tension but will not affect the strength in compression.

(13) Factor of safety :

The ratio of the maximum stress that a structural part or other piece of material can withstand to the maximum stress estimated for it in the use for which it is designed called factor of safety

(14) Percentage elongation :

Percent elongation signifies the ability of a element or compound to stretch upto its breaking point it is measured by dividing the change in length (upto the breaking point) by the original length then multiplying by 100. materials with a higher percentage elongation can stretch more before breaking.

(15) Percentage reduction in area :

Reduction of area is proportional reduction of the cross sectional area of a tensile test piece at the place of fracture measured after fracture.

Percent reduction of area (RA) =

$$= \frac{\text{Area of original cross section} - \text{minimum final area}}{\text{Area of original cross section}}$$

$$= \frac{A_0 - A_{min}}{A_0} = \frac{\text{Decrease in area}}{\text{original area}}$$

$$= \frac{\text{square inches}}{\text{square inches}} \times 100$$

The reduction of area is reported as additional information characteristics of the material.

(16) Stresses in bars of varying (stepped) sections :

When a bar is made up of different lengths having different cross-sectional areas, and is subjected to an axial force P, as shown in

figure, then the total deformation of the bar,

$$\delta l = \delta l_1 + \delta l_2 + \delta l_3 + \cdots$$

$$= \frac{P\,l_1}{A_1 E} + \frac{P\,l_2}{A_2 E} + \frac{P\,l_3}{A_3 E} + \cdots \qquad \left[\because\ \delta l = \frac{Pl}{AE}\right]$$

$$= \frac{P}{E}\left(\frac{l_1}{A_1} + \frac{l_2}{A_2} + \frac{l_3}{A_3} + \cdots\right)$$

(ii) Stresses in bars of uniformly [redacted] tapering (linearly varying) circular section :

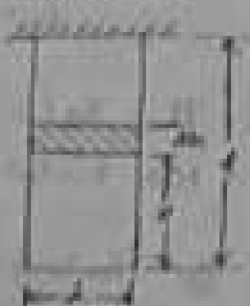
When a bar of uniformly tapering circular section is subjected to an axial force P, as shown in figure, then the elongation of the bar,

$$\delta l = \frac{4Pl}{\pi E d_1 d_2}$$

Note: If the bar had been uniform diameter & throughout, then, $\delta l = \frac{4Pl}{\pi E d^2} = \frac{Pl}{\frac{\pi}{4} d^2 E} = \frac{Pl}{AE}$

(iii) Stress in bar due to its own weight :

Consider a bar of length l and diameter (d) rigidly fixed at the upper end and hanging freely as shown in figure. The stress at any section of a bar due to its own weight is directly proportional to the distance from the free end. Therefore, stress at a distance x from the free end $= w \cdot x$ $\quad (\because P = w \cdot A \cdot x)$



and total elongation of the bar,

$$\delta l = \frac{w l^2}{2E}$$

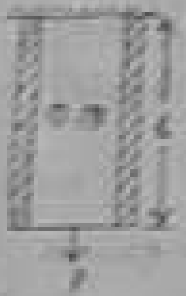
where w = weight per unit volume of the bar

NOTE: When a conical bar of length (l) and base diameter (d) is rigidly fixed with its base diameter at the upper end and is hanging freely then the total elongation of the bar due to its own weight is given by

$$\delta l = \frac{wl}{2E}$$

8 Stresses in composite bar :

A composite bar may be defined as a bar made up of two or more different materials, joined together in such a manner that the system extends or contracts as one unit, equally when subjected to tension or compression.



Consider a composite bar made up of two different materials as shown in figure.

Load shared by bar 1, $P_1 = P \times \frac{A_1E_1}{A_1E_1 + A_2E_2}$

and load shared by bar 2, $P_2 = P \times \frac{A_2E_2}{A_1E_1 + A_2E_2}$

Since the elongation in both the bars is same ($\delta l_1 = \delta l_2$), therefore

$$\frac{P_1 l}{A_1E_1} = \frac{P_2 l}{A_2E_2} \quad \text{or,} \quad \frac{\sigma_1}{E_1} = \frac{\sigma_2}{E_2}$$

$$\sigma_1 = \frac{E_1}{E_2} \times \sigma_2 \quad \text{and} \quad \sigma_2 = \frac{E_2}{E_1} \times \sigma_1$$

The ratio E_1/E_2 is known as modular ratio of the two materials.

9 Thermal stress (or temperature stress):

Whenever there is some increase or decrease in the temperature of a body, it causes the body to expand or contract. A little consideration will show that if the body is allowed to expand or contract freely with the rise or fall of the temperature, no stresses are induced in the body. But if the deformation of the body is prevented, some stresses are induced in the body. Such stresses are known as thermal stresses, and the corresponding strains are called thermal strains.

When a bar of length (l) is subjected to an increase in temp (t), then the increase in length when the bar is free to expand is given by,

$$\delta l = l \alpha t$$

where, α = coefficient of linear expansion

If the ends of the bar are fixed to rigid supports so that its expansion is prevented, then compressive thermal strain induced in the bar,

$$\varepsilon = \delta l / l = l\alpha t / l = \alpha t$$

and thermal stress $= \varepsilon E = \alpha t E$

Note: If the supports yield by an amount equal to δ, then the actual expansion that has taken place,

$$\delta l = l\alpha t - \delta$$

and strain, $\varepsilon = \frac{\delta l}{l} = \frac{l\alpha t - \delta}{l} = \left(\alpha t - \frac{\delta}{l}\right)$

$\therefore$ Stress, $\sigma = \varepsilon E = \left(\alpha t - \frac{\delta}{l}\right) E$

(8) A aluminium alloy bar, fixed at its both ends, is heated through 20 K. Find the stress developed in the bar. Take modulus of elasticity and coefficient of linear expansion for the bar material as 80 GPa and 24×10^{-6}/K respectively.

Solution: Given: Increase in temp $(t) = 20$ K,

$E = 80$ GPa $= 80 \times 10^3$ N/mm^2 and $\alpha = 24 \times 10^{-6}$/K

$\therefore$ Thermal stress developed in the bar,

$$\sigma_{th} = \alpha t E$$

$$= (24 \times 10^{-6}) \times 20 \times (80 \times 10^3) \text{ N/mm}^2$$

$$= 38.4 \text{ N/mm}^2$$

$$= 38.4 \text{ MPa} \quad \text{Ans.}$$

(1) A brass rod 3 m long is fixed at both its ends. If the thermal stress is not to exceed 76.5 MPa, calculate the temp through which the rod should be heated. Take the values of α and E as $17 \times 10^{-6}/K$ and 90 GPa respectively.

<u>Solution:</u> Given: Length (l) = 3 m, Maximum thermal stress $\sigma_{max} = 76.5$ MPa $= 76.5\ N/mm^2$, $\alpha = 17 \times 10^{-6}/K$ and

$E = 90$ GPa $= 90 \times 10^3\ N/mm^2$

Let t = Temp through which the rod should be heated in K.

Maximum stress in the rod (σ_{max})

$$76.5 = \alpha . t . E = (17 \times 10^{-6}) \times t \times (90 \times 10^3)$$

$$= 1.53\, t$$

$$\therefore\ t = \frac{76.5}{1.53} = 50\ K$$ Ans.

(2) Two parallel walls 6 m apart are stayed together by a steel rod 25 mm diameter passing through metal plates and nuts at each end. The nuts are tightened home, when the rod is at a temp of 100°C. Determine the stress in the rod, when the temp falls down to 60°C, if (i) the ends do not yield and (ii) the ends yield by 1 mm.

Take E = 200 GPa and $\alpha = 12 \times 10^{-6}/°C$

<u>Solution:</u> Given length (l) = 6 m = 6×10^3 mm, Dia (d) = 25 mm, Decrease in temp (t) = 100° − 60° = 40°C, Amount of yield in ends (δ) = 1 mm, $E = 200\ GPa = 200 \times 10^3\ N/mm^2$, $\alpha = 12 \times 10^{-6}/°C$

(i) Stress in the rod when the ends do not yield

$$\sigma_1 = \alpha . t . E = (12 \times 10^{-6}) \times 40 \times (200 \times 10^3)\ N/mm^2$$

$$= 96\ N/mm^2 = 96\ MPa$$ Ans.

(ii) Stress in rod when the ends yield,

$$\sigma_2 = \left[\alpha t - \frac{\delta}{l}\right] E = \left[(12 \times 10^{-6}) \times 40 - \frac{1}{6 \times 10^3}\right] \times 200 \times 10^3\ N/mm^2$$

$$= 62.6\ N/mm^2 = 62.6\ MPa$$ Ans.

(1) Q.11 A reinforced concrete circular section of 50000 mm² cross sectional area carrying 6 reinforcing bars whose total area is 500 mm². Find the safe load, the column can carry, if the concrete is not to be stressed more than 3.5 MPa. Take modular ratio of steel and concrete as 18.

__Solution__: Given Area of column = 50,000 mm², No of reinforcing bars = 6, Total area of bars (A_s) = 500 mm², max stress in concrete (σ_c) = 3.5 MPa = 3.5 N/mm² and modular ratio (E_s/E_c) = 18

∴ Area of concrete, A_c = 50000 − 500 = 49,500 mm²

and stress in steel,

$$\sigma_s = \frac{E_s}{E_c} \times \sigma_c = 18 \times 3.5 = 63 \text{ N/mm}^2$$

∴ Safe load the column can carry,

$$P = (\sigma_s \cdot A_s) + (\sigma_c \cdot A_c)$$
$$= (63 \times 500) + (3.5 \times 49,500) \text{ N}$$
$$= 204,750 \text{ N} = 204.75 \text{ kN}$$ __Ans__:

(2) A reinforced concrete column 500 mm × 500 mm in section is reinforced with 4 steel bars of 25 mm diameter, one in each corner. The column is carrying a load of 1000 kN. Find the stress in the concrete & steel bars. Take E for steel = 210 GPa and E for concrete = 14 GPa.

__Solution__: Given: Area of column = 500×500 = 2,50,000 mm², No of steel bars (n) = 4, Dia of steel bars (d) = 25 mm, load on column (P) = 1000 kN = 1000×10³ N, E_s = 210 GPa and E_c = 14 GPa.

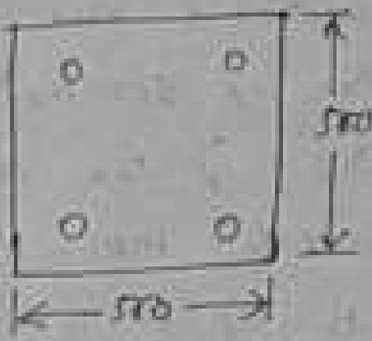
Let, σ_s = stress in steel and
σ_c = stress in concrete

Area of steel bars,

$A_s = 4 \times \frac{\pi}{4} \times d^2 \ mm^2$

$= 4 \times \frac{\pi}{4} \times 25^2$

$= 1963 \ mm^2$



∴ Area of concrete,

$A_c = 250\,000 - 1963 \ mm^2$

$= 248\,037 \ mm^2$

∴ Stress in steel,

$\sigma_s = \frac{E_s}{E_c} \times \sigma_c = \frac{210}{14} \times \sigma_c = 15\,\sigma_c$

And total load (P),

$1000 \times 10^3 = (\sigma_s A_s) + (\sigma_c A_c)$

$= (15 \times \sigma_c \times 1963) + (\sigma_c \times 248\,037)$

$= 277\,482\,\sigma_c$

or, $\sigma_c = \frac{1000 \times 10^3}{277\,482} = 3.6 \ N/mm^2$

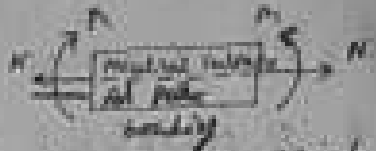
$= 3.6$ MPa **Ans.**

and $\sigma_s = 15\,\sigma_c = 15 \times 3.6 = 54$ MPa **Ans.**

Shear force and bending moment

Types of loads and beams :

Axial load :



Consider the case where a beam has both an axial load and a bending moment. If the axial load passes through the neutral axis passing for pure bending, the axial load will not contribute to additional bending and one can consider the bending as a linear superposition of pure bending and uniform extension.

Transverse load :

Transverse loading is a load applied vertically to the plane of the longitudinal axis of a configuration, such as a wind load. It causes the material to bend and rebound from its original position, with inner tensile and compressive straining associated with the change in curvature of the material. It is known as crosswise force.

Concentrated point load :

A load acting at a point of beam is known as a concentrated or point load.

Uniformly distributed load :

A load which is spread over a beam in such a manner that each unit length is loaded to the same extent, is known as uniformly distributed load (UDL).

Uniformly varying load :

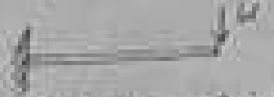
A load which is spread over a beam in such a manner that it varies uniformly on each unit length is known as uniformly varying load. Sometimes the load is zero at one end and increases

uniformly to the other end a load is known as triangular load.

(10) Types of beams:

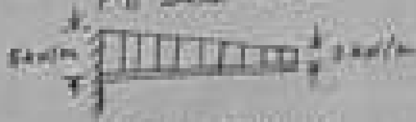
(1) Cantilever beam:

A beam fixed at one end and free at other end is known as cantilever beam.



C.B with a point load

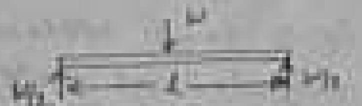
C.B with U.D.L



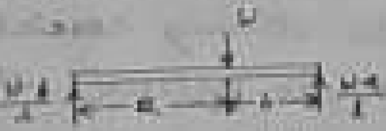
C.B with U.V.L

(2) Simply supported beam:

A beam supported at its both side is known as simply supported beam.



S.S.B with point load at its middle point



with an eccentric point load

with a U.D.L

(3) Overhanging beam:

A beam having its end portion extended beyond the support, is known as overhanging beam. A beam may be overhanging on one side or on both sides.

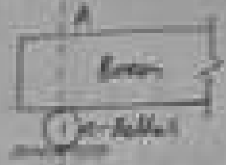
(4) Fixed beam:

A beam whose both ends are fixed, is known as fixed beam.

(5) Continuous beam:

A beam supported on more than two supports is known as continuous beam.

① Types of supports:



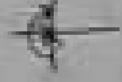
Roller supports:

Roller supports are free to rotate and translate along the surface upon which the roller rests. The surface may be horizontal, vertical or sloped at any angle. Roller supports are commonly located at one end of long bridges in the form of bearing pads. These supports allows bridge structure to expand and contract with temperature changes without fracture the supports at the base.

Hinge supports:

The hinge support is capable of resisting forces acting in any direction of the plane. This supports does not provide any resistance to rotation. Hinge support may also be used in three hinged [illegible] bridges at the base supports while at the central internal hinged is introduced. It is also used in doors to produce only rotation in a door. Hinge supports reduces sensitivity to earthquake.

Fixed support:



Fixed support can resist vertical and as well as moment since they restrain both rotation and translation. They are also known as rigid support for the stability of a structure there should be one fixed support. A flagpole at concrete base is common example of fixed support. [illegible] welded joints in steel Similarly all the riveted and structure are the examples of fixed supports. Riveted connection are not very much common due to introduction of bolted joints.

(1) Shear force and bending moment in beams :

(2) Shear force :

The shear force is the unbalanced vertical force, therefore it tends to slide one portion of the beam upwards or downwards with respect to the other. In other words shear force may be defined as the algebraic sum of all the forces on either side of the section.

(3) Sign convention of shear force diagram :

(a) Positive S.F (b) Negative S.F

The shear force is said to be positive at a section, when the left hand portion tends to slide downwards or the right hand portion slide upwards as shown in figure(a), or in other words all the downwards forces to the left of the section cause positive shear and those acting upwards cause negative shear.

Similarly, the shear force is said to be negative at a section when the left hand portion tends to slide upwards or right hand portion tends to slide downwards as shown in fig (b) or in other words, all the upwards forces to the left of the section cause negative shear and those acting downwards cause positive shear in fig (b).

(4) Bending moment :

The bending moment (B.M) at the cross section of a beam may be defined as the algebraic sum of all the moments of the forces on either side of the portion

→ P.T.O

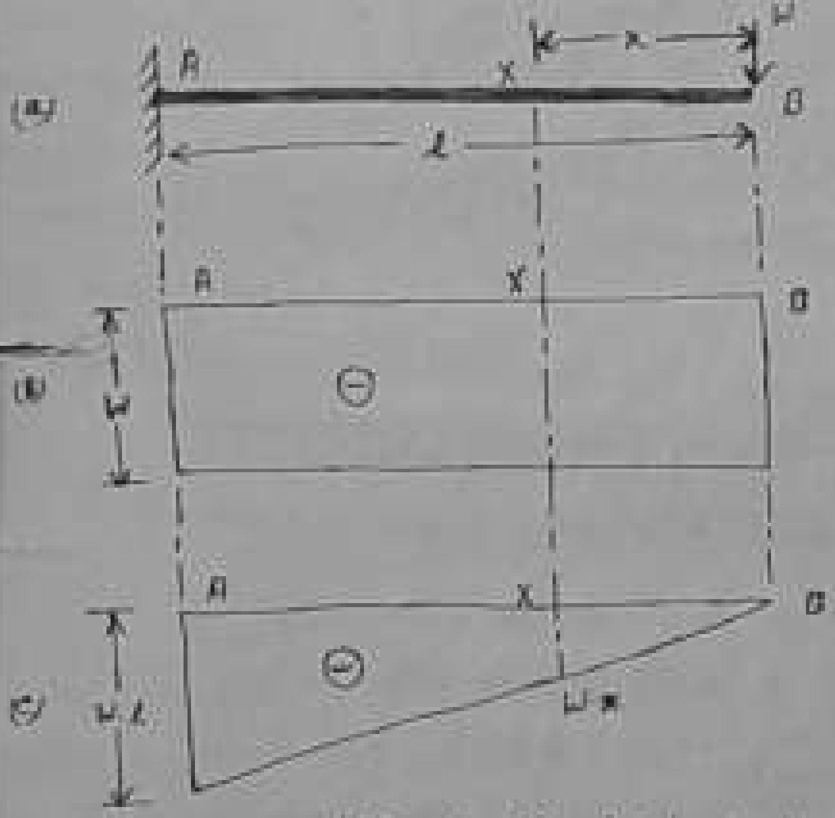
(ii) Cantilever with a point load at its free end :

Consider cantilever AB of length l and carrying a point load W at its free end B as shown in figure (a). We know that shear force at any section X, at a distance x from the free end, is equal to the total unbalanced vertical force, i.e.,

$F_x = -W$ (minus sign due to right downward)

and bending moment at this section,

$M_x = -W.x$ (minus sign due to sagging)



Cantilever with point load

Thus from the equation of shear force, we see that the shear force is constant and is equal to -W at all sections between B and A. And from the bending moment equation, we see that the bending moment is zero at B (where x=0) and increases by a straight line law to -Wl at (where x=l). Now draw the

* Bending moment sign convention :

(a) Positive BM (b) Negative BM

As spacing, where the bending moment is such that it tends to bend the beam at that point to a curvature having concavity at the top as shown in fig (a) is taken as positive on the other hand, where the bending moment is such that it tends to bend the beam at that point to a curvature having convexity at the top, as shown in fig (b) is taken as negative.
The positive bending moment is often called sagging moment and negative as hogging moment.

A little consideration will ~~show that~~ the bending moment is said to be positive, at a section when it is acting in an anticlockwise direction to the right and negative when acting in a clockwise direction, on the other hand, the bending moment is said to be negative when it is acting in a clockwise direction to the left and positive when it is acting in an anticlockwise direction.

diagrams as shown ...

PROB (81) Draw shear force and bending moment diagrams for a cantilever beam of span 1.5 m carrying point loads as shown in fig (a).

Solution: Given: Span (L) = 1.5 m, Point load at B (W_1) = 1.5 kN and Point load at C (W_2) = 2 kN

The shear force diagram is shown in fig (b) and the values are tabulated here:

$$F_B = -W_1 = -1.5\ kN$$

$$F_C = -(1.5 + W_2) = -(1.5+2) = -3.5\ kN$$

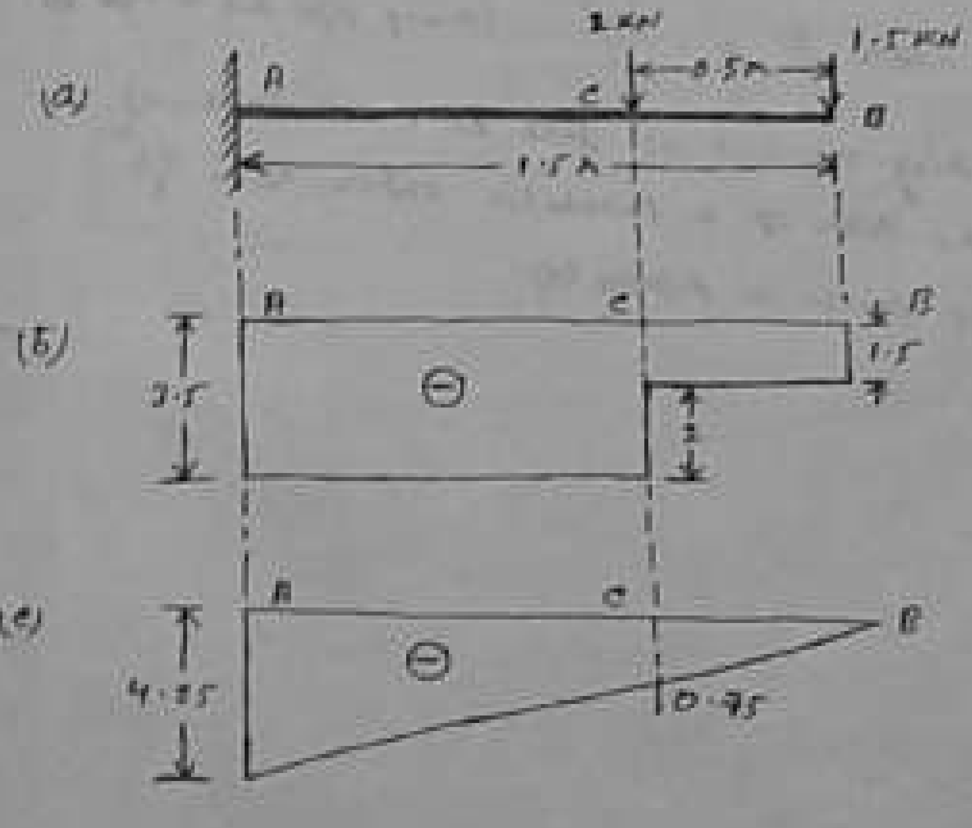
$$F_A = -3.5\ kN$$

The Bending moment diagram is shown in fig (c) and the values are tabulated here:

$$M_B = 0$$

$$M_C = -[1.5 \times 0.5] = -0.75\ kN\text{-}m$$

$$M_A = -[(1.5 \times 1.5) + (2 \times 1)] = -4.25\ kN\text{-}m$$



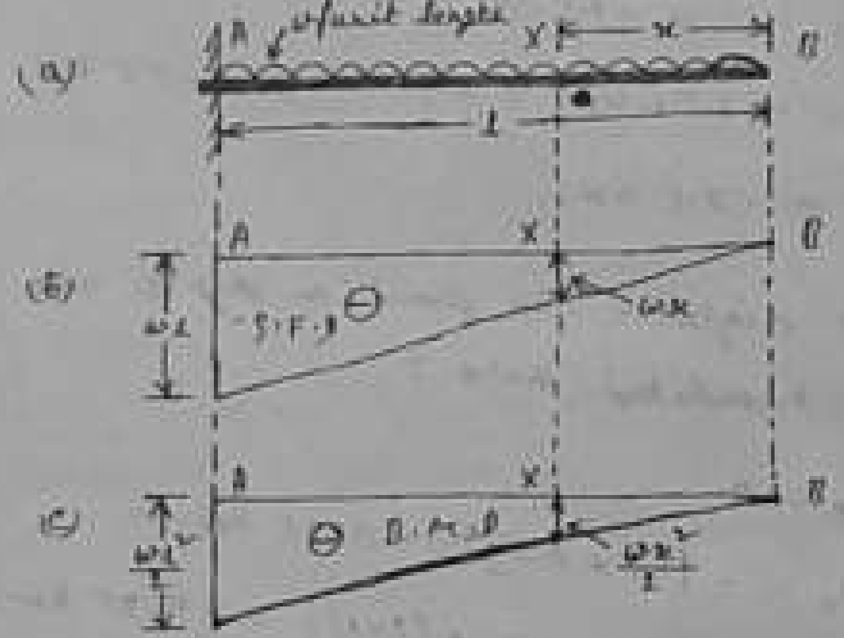
Cantilever with a uniformly distributed load:

Consider a cantilever AB of length l and carrying a uniformly distributed load of w per unit length, over the entire length of the cantilever as shown in figure (a).

We know that shear force at any section X, at a distance x from B,

$$F_x = -w \cdot x \quad \text{(minus sign due to right downwards)}$$

Thus we see that shear force is zero at B (where $x=0$) and increases by a straight line law to $-wl$ at A as shown in figure (b).



We also know that bending moment at X,

$$M_x = -w.x \cdot \frac{x}{2} = -\frac{wx^2}{2} \quad \text{(minus sign due to hogging)}$$

Thus the bending moment is zero at B (where $x=0$) and increases in the form of a parabolic curve to $-\frac{wl^2}{2}$ at A (where $x=l$) as shown in figure (c).

Prob: (3) A cantilever beam AB, 2m long carries a uniformly distributed load of 1.5 kN/m over a length of 1.6 m from the free end. Draw shear force and bending moment diagrams for the beam.

Solution: Given: span (l) = 2m, uniformly distributed load (w) = 1.5 kN/m and length of the cantilever CB carrying load (a) = 1.6 m.

The shear force diagram is shown in figure (1) and the values are tabulated here:

$$F_B = 0$$

$$F_C = -w \cdot a = -1.5 \times 1.6 = -2.4 \text{ kN}$$

$$F_A = -2.4 \text{ kN}$$

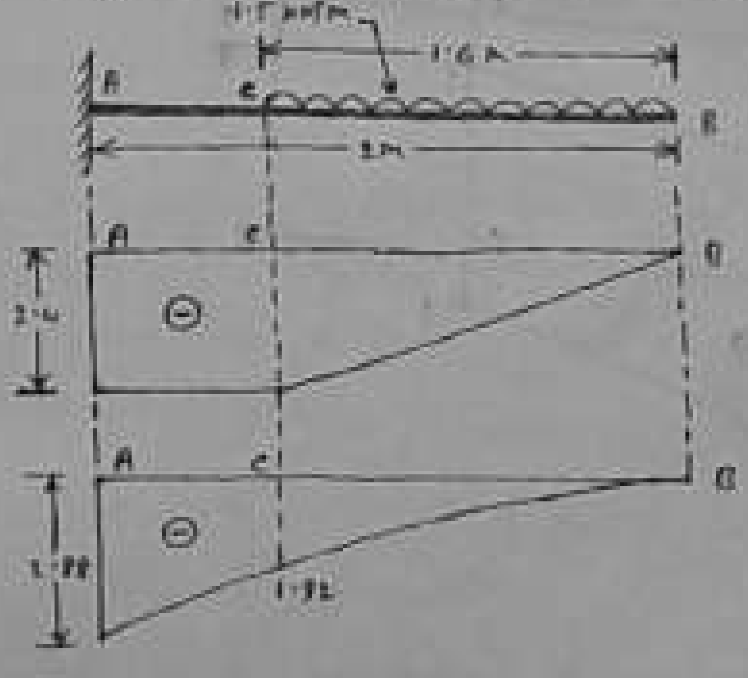
The bending moment diagram is shown in figure (2) and the values are tabulated here:

$$M_B = 0$$

$$M_C = -\frac{wa^2}{2} = -\frac{1.5 \times (1.6)^2}{2} = -1.92 \text{ kN-m}$$

$$M_A = -\left[(1.5 \times 1.6)\left(0.4 + \frac{1.6}{2}\right)\right] = -2.88 \text{ kN-m}$$

Note: The bending moment at A is the moment of the load between C and B (equal to 1.5 × 1.6 = 2.4 kN) about A. The distance between the centre of the load and A is $0.4 + \frac{1.6}{2} = 1.2$ m.



Prob: (03) A cantilever beam of 1.5 m span is loaded as shown in figure (a). Draw the shear force and bending moment diagrams.

Solution: Given: Span (L) = 1.5 m, Point load at B (W) = 2 kN, uniformly distributed load (w) = 1 kN/m and length of the cantilever AC carrying the load (a) = 1 m.

The shear force diagram is shown in figure (b) and the values are tabulated here:

$$F_B = -W = -2 \text{ kN}$$

$$F_C = -2 \text{ kN}$$

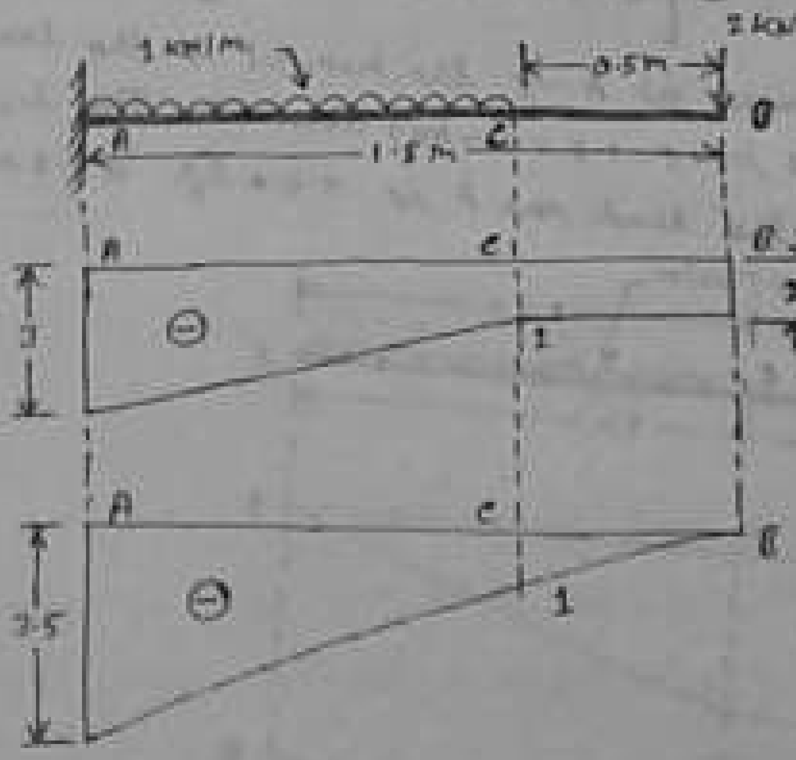
$$F_A = -[2 + (1 \times 1)] = -3 \text{ kN}$$

The bending moment diagram is shown in figure (c) and the values are tabulated here:

$$M_B = 0$$

$$M_C = -[2 \times 0.5] = -1 \text{ kN-m}$$

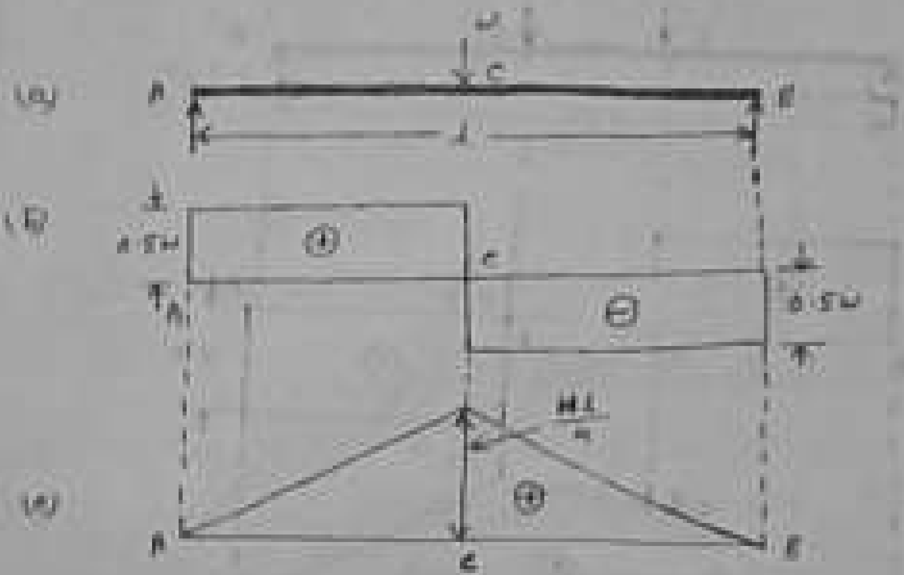
$$M_A = -\left[(2 \times 1.5) + (1 \times 1) \times \tfrac{1}{2}\right] = -3.5 \text{ kN-m}$$



(ii) Simply supported beam with a point load at its mid-point:

Consider a simply supported beam AB of span l and carrying a point load W at its mid-point C as shown in figure (a). Since the load is at the mid point of the beam, therefore the reaction at the support A,

$$R_A = R_B = 0.5W$$



Thus we see that the shear force at any at any section between A and C (i.e. the point just before the load W) is constant and is equal to the unbalanced vertical force +0.5W. Shear force at any section between C and B (just after the load W) is also constant and is equal to the unbalanced vertical force −0.5W as shown in fig (b).

We also see that the bending moment at A and B is zero. It increases by a straight line law and is maximum at centre of beam, where shear force changes sign as shown in figure (c).

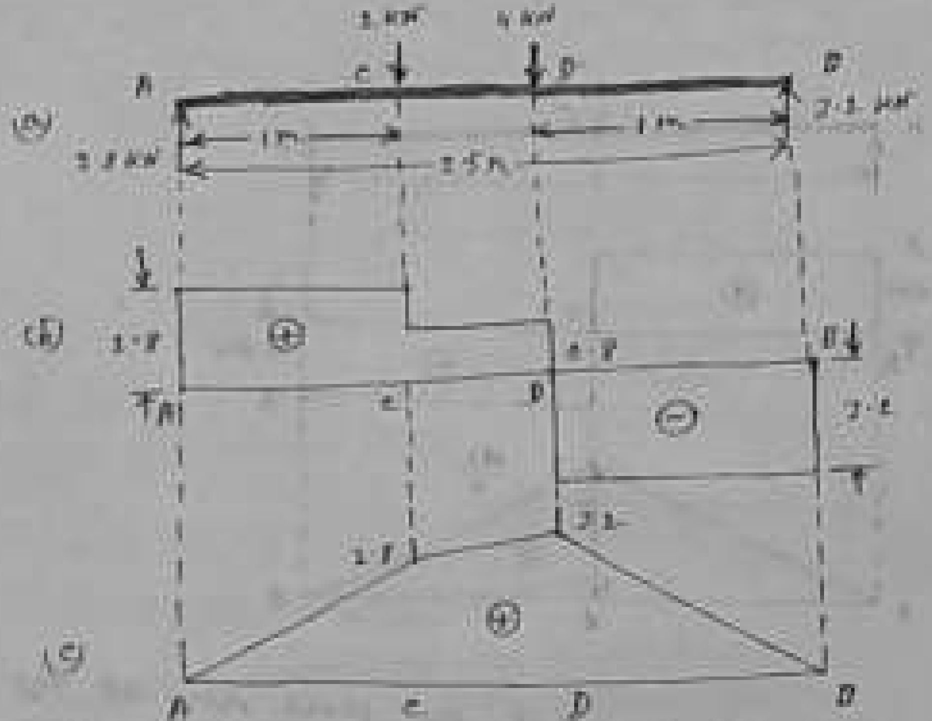
Therefore bending moment at C,

$$M_C = \frac{W}{2} \times \frac{l}{2} = \frac{Wl}{4} \quad \text{... (Plus sign due to sagging)}$$

Note: If the point load does not act at mid point of the beam, then the two reactions are obtained and the diagrams are drawn as usual.

Prob-(4) A simply supported beam AB of span 2.5 m is carrying two point loads as shown in figure. Draw the shear force and bending moment diagrams for the beam.

Solution: Given: Span (L) = 2.5 m, Point load at C (W_1) = 2 kN and Point load at D (W_2) = 4 kN.



First of all let us find out the reactions R_A and R_B. Taking moments about A and equating the same,

$$R_B \times 2.5 = (2 \times 1) + (4 \times 1.5) = 8$$

$$\text{or, } R_B = 8/2.5 = 3.2 \text{ kN}$$

$$\text{and } R_A = (2+4) - 3.2 = 2.8 \text{ kN}$$

The shear force diagram is shown in figure (b) and the values are tabulated here:

$$F_A = +R_A = 2.8 \text{ kN}$$

$$F_C = +2.8 - 2 = 0.8 \text{ kN}$$

$$F_D = 0.8 - 4 = -3.2 \text{ kN}$$

$$F_B = -3.2 \text{ kN}$$

The bending moment diagram is shown in figure.

$M_A = 0$

$M_C = 2.8 \times 1 = 2.8$ kN-m

$M_D = 3.2 \times 1 = 3.2$ kN-m

$M_B = 0$

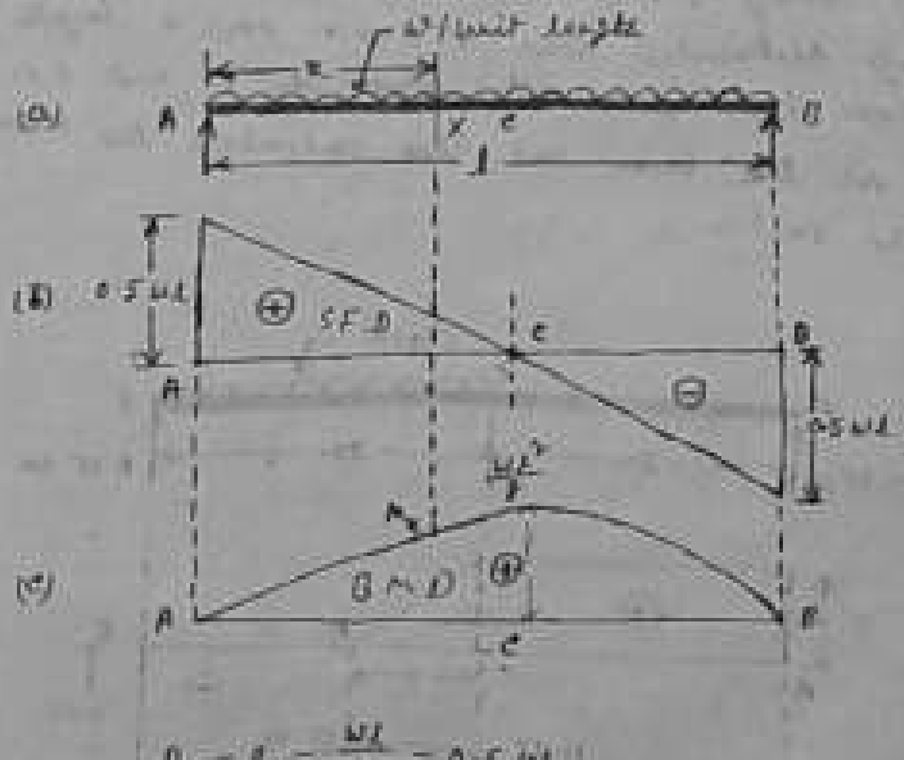
Note: The value of M_D may also be found out from the reaction R_A,

$M_D = (2.8 \times 1.5) - (2 \times 0.5)$

$= 4.2 - 1.0 = 3.2$ kN-m **Ans:**

(iii) Simply supported beam with a uniformly distributed load:

Consider a simply supported beam AB of length l and carrying a uniformly distributed load of w per unit length as shown in figure. Since the load is uniformly distributed over the entire length of the beam, therefore the reactions at the supports A, B



$R_A = R_B = \frac{wl}{2} = 0.5\,wl$

We know that shear force at any section X at a distance x from A,

$F_x = R_A - wx = 0.5\,wl - wx$

Solution: Given: Span (l) = 6 m, uniformly distributed load (w) = 5 kN/m and length of the beam CB carrying load (a) = 3 m.

First of all, let us find out the reactions R_A and R_B. Taking moment about A and equating the same,

$$R_B \times 6 = (5 \times 3) \times 4.5 = 67.5$$

$$R_B = \frac{67.5}{6} = 11.25 \text{ kN}$$

and $R_A = (5 \times 3) - 11.25 = 3.75$ kN

Shear force diagram,

The S.F diagram is shown in figure (b) and the values are tabulated here:

$$F_A = +R_A = +3.75 \text{ kN}$$

$$F_C = +3.75 \text{ kN}$$

$$F_B = +3.75 - (5 \times 3) = -11.25 \text{ kN}$$

The bending moment diagram, is shown in figure (c) and the values are tabulated here.

$$M_A = 0$$

$$M_C = 3.75 \times 3 = 11.25 \text{ kN}$$

$$M_B = 0$$

The maximum bending moment will occur at M, where the shear force changes sign. Let x be the distance between C and M. From the geometry of the figure between C and B, we find that,

$$\frac{x}{3.75} = \frac{3-x}{11.25} \quad \text{or, } 11.25x = 11.25 - 3.75x$$

or, $15x = 11.25$ or, $x = 11.25/15 = 0.75$ m

$$\therefore M_M = 3.75 \times (3 + 0.75) - 5 \times \frac{0.75}{2} = 11.66 \text{ kN-m} \quad \underline{\text{Ans.}}$$

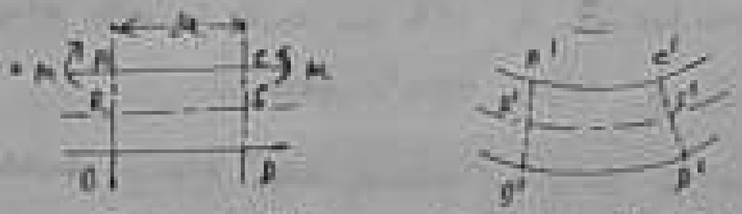
(1) Stresses in beams and shafts :

(i) Bending stress in beam :

The bending moment at a section tends to bend or deflect the beam and the internal stresses resist its bending. The process of bending stops, when every section sets up full resistance to the bending moment. The resistance offered by the internal stresses, to the bending, is called bending stress.

(ii) Theory of simple bending :

Consider a small length of a simply supported beam subjected to a bending moment as shown in fig (a). Now consider two sections AB and CD, which are normal to the axis of the beam RS. Due to action of the bending moment, the beam as a whole will bend as shown in fig (b).



(a) Before bending (b) After bending

Since we are considering a small length of dx of the beam, therefore the curvature of the beam in this length, is taken to be circular (A little consideration will show that all the layers of the beam, which were originally of the same length do not remain of the same length anymore.) The top layer of the beam has suffered compression and reduced to A'C'. As we proceed towards the lower layers of the beam, we find that the layers have no doubt suffered compression, but to lesser degree, until we come across the layer RS, which has suffered no change in its length, though bent into R'S'. If we further proceed towards the lower layers, we find the layers have suffered tension, as a result of which the layers the stretched. The amount of extension increases as we proceed lower, untill we come across the lowermost layer BD which has been stretched to B'D'.

Now we see that the layers above have been compressed and those below RS have been stretched. The amount by which layer is compressed or stretched, depends upon the position of the layer with reference to RS. This layer RS which is neither compressed nor stretched, is known as neutral plane or neutral layer. This theory of bending is called theory of simple bending.

(i) Assumptions in the the theory of simple bending:

(1) The material of the beam is perfectly homogeneous (of the same kind of throughout) and isotropic (or equal elastic properties in all directions).

(2) The beam material is stressed within its elastic limit and thus obeys Hooke's law.

(3) The transverse section which are plane, before bending, remains plane after bending also.

(4) Each layer of the beam is free to expand or contract, independently, of the layer above or below it.

(5) The value of E (Young's modulus of elasticity) is the same in tension and compression.

(6) The beam is in equilibrium, there is no resultant pull or push in the beam section.

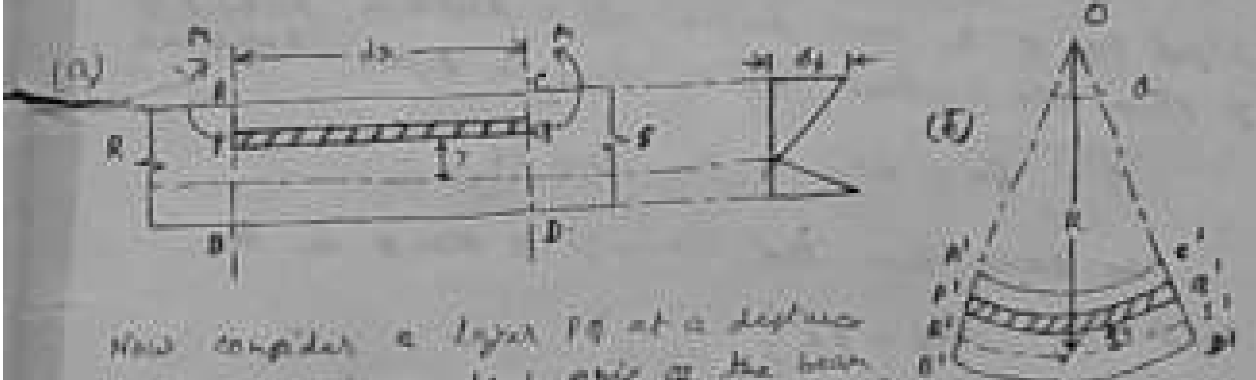
(17) Moment of resistance:

The compressive stresses on one side of the neutral axis and tensile stresses on the other side, therefore these stresses form a couple whose moment must be equal to the external moment M. The moment of this couple which resist the external bending moment, is known as moment of resistance.

(18) Derivation of bending equation (flexure):

Consider a small length dx of a beam subjected to a bending moment as shown in fig (a). As a result of this moment, let this small length of beam bend into an arc of a circle with O as centre as shown in fig (b).

Let, M = moment acting at the beam,
θ = Angle subtended at the centre by the arc
and R = Radius of curvature of the beam.



Now consider a layer PQ at a distance y from RS the neutral axis of the beam. Let this layer be compressed to P'Q' after bending as shown in fig (b).

We know that decrease in length of this layer,

$$\delta l = PQ - P'Q'$$

$$\text{Strain } \varepsilon = \frac{\delta l}{\text{original length}} = \frac{PQ - P'Q'}{PQ}$$

Now from the geometry of the curved beam, we find that the two sections $OP'Q'$ and $OR'S'$ are similar

$$\frac{P'Q'}{R'S'} = \frac{R-y}{R} \quad \text{or,} \quad 1 - \frac{P'Q'}{R'S'} = 1 - \frac{R-y}{R}$$

$$\text{or,} \quad \frac{R'S' - P'Q'}{R'S'} = \frac{y}{R}$$

or, $\frac{PQ - P'Q'}{PQ} = \frac{y}{R}$ $(PQ = P'Q' = \text{neutral axis})$

or, $\varepsilon = \frac{y}{R}$ $\left(\because \varepsilon = \frac{PQ - P'Q'}{PQ}\right)$

The strain (ε) of a layer is proportional to the distance from the neutral axis.

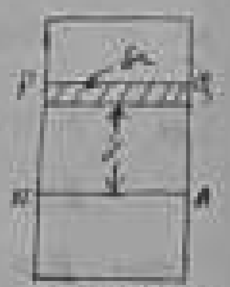
We know that the bending stress,

$$\sigma_b = \text{strain} \times \text{elasticity} = \varepsilon \times E$$

$$= \frac{y}{R} \times E = y \times \frac{E}{R}$$

Since E and R are constants in this expression, therefore the stress at any point is directly proportional to y, the distance of the point from the neutral axis.

$$\therefore \frac{\sigma_b}{y} = \frac{E}{R}$$



Moment of resistance

Consider a section of the beam as shown in figure. Let NA be the neutral axis of the section. Now consider a small layer PQ of the beam section at a distance y from the neutral axis as shown in figure.

Let δa = Area of the layer PQ

We have seen the intensity of stress in the layer PQ, $\sigma = y \times \frac{E}{R}$

Total stress in the layer PQ $= y \times \frac{E}{R} \times \delta a$

and moment of this total stress about the neutral axis,

$$= y \times \frac{E}{R} \times \delta a \times y = \frac{E}{R} y^2 \delta a$$

The algebraic sum of all such moments about the neutral axis must be equal to M. Therefore,

$$M = \sum \frac{E}{R} y^2 \delta a = \frac{E}{R} \sum y^2 \delta a$$

The expression $\sum y^2 \delta a$ represents the moment of inertia of the area of the whole section about neutral axis.

Therefore,

$$M = \frac{E}{R} \times I$$

or, $$\frac{M}{I} = \frac{E}{R}$$

we have already seen that,

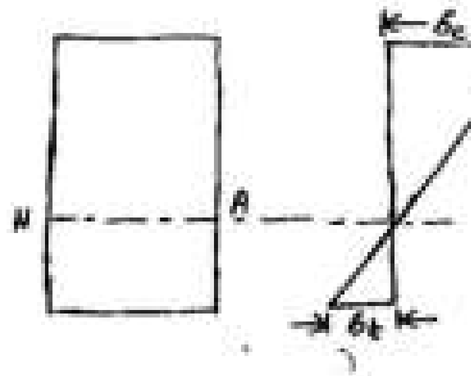
$$\frac{f}{y} = \frac{E}{R}$$

$$\boxed{\frac{M}{I} = \frac{f}{y} = \frac{E}{R}}$$

It is the most important equation in the theory of simple bending, which gives us relation between various characteristics of beam.

(11) Bending stress distribution:

We know that there is no stress at a the neutral axis. In a simply supported beam, there is a compressive stress above the neutral axis and tensile stress below it. We also know that the stress at a point is directly proportional to its distance from the neutral axis. If we plot the stresses in a simply supported beam section, we shall get a figure as shown in fig below.



The maximum stress (either compressive or tensile) takes place at outmost layer. Or in other words, while obtaining maximum bending stress at a section, the value of y is taken as maximum.

(iii) Modulus of section :

We know that the relation bet. bending stress and the bending stress on the extreme fibre of a section, $\frac{M}{I} = \frac{\sigma}{y}$ or, $M = \sigma \times \frac{I}{y}$

From this relation, we find that the stress in a fibre is proportional to its distance from the c.g. If y_{max} is the distance between c.g of the section and the extreme fibre of the stress, then

$$M = \sigma_{max} \times \frac{I}{y_{max}} = \sigma_{max} \times Z$$

where $Z = \frac{I}{y_{max}}$. The term 'Z' is known as modulus of section or section modulus.

If the section of beam is, is symmetrical, its centre of gravity and hence the neutral axis we lie at the middle of its depth.

For Rectangular section M.I about an axis through its c.g. $I = \frac{bd^3}{12}$

$\therefore$ Modulus of section $Z = \frac{I}{y} = \frac{bd^3}{12} \times \frac{2}{d} = \frac{bd^2}{6}$ $\left[\because y = \frac{d}{2}\right]$

For circular section M.I about an axis through its c.g,

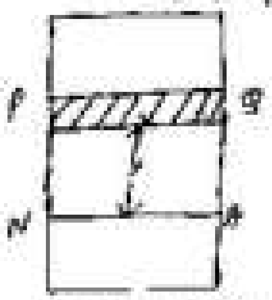
$$I = \frac{\pi}{64} d^4$$

$\therefore$ modulus of section $Z = \frac{I}{y} = \frac{\pi}{64} d^4 \times \frac{2}{d} = \frac{\pi}{32} d^3$ $\left[\because y = \frac{d}{2}\right]$

(iv) Position of neutral axis and centroidal axis :

The line of intersection of the neutral layer, with any normal cross section of a beam, is known as neutral axis of that section. One side of the neutral axis are compressive and other side are tensile stresses. At the neutral axis there is not stress of any kind.

Consider a section of the beam as shown in figure. Let be the neutral axis of the section. Consider a small layer PQ of the beam section at a distance from the neutral axis as shown in figure.



Neutral axis

Let, δa = Area of layer PQ

The intensity of stress in the layer PQ,

$$f = y \times \frac{E}{R}$$

$\therefore$ Total stress on the layer PQ

$$= \text{Intensity of stress} \times \text{Area}$$

$$= y \times \frac{E}{R} \times \delta a$$

and total stress of the section

$$= \sum y \times \frac{E}{R} \times \delta a = \frac{E}{R} \sum y \cdot \delta a$$

Since the section is in equilibrium, therefore total stress from top to bottom, must be equal to zero.

$\therefore$ [redacted] $\frac{E}{R} \sum y \cdot \delta a = 0$

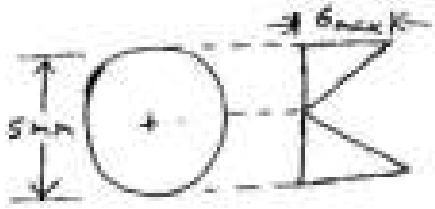
or, $\sum y \cdot \delta a = 0$ $\left[\because \frac{E}{R} \text{ cannot be equal to zero}\right]$

A little consideration will show that $y \times \delta a$ is the moment of the area about the neutral axis and $\sum y \cdot \delta a$ is the moment of the entire area of the cross section about the neutral axis. So the neutral axis of the section will be so located that moment of entire area about the axis is zero. We know that moment of area about an axis passing through its central axis of a section always passes through its centroid. Thus to locate the neutral axis of [redacted] a section, first find out the centroid of the section and draw a line passing through this centroid and normal to the plane of bending. This line will be neutral axis of the section.

(ii) Prob (3) A steel wire of 5 mm diameter is bent into a circular shape of 5 m radius. Determine the maximum stress induced in the wire. Take E = 200 GPa.

Solution: Given: Dia of steel wire (d) = 5 mm, Radius of circular shape (R) = 5 m = 5×10^3 mm and modulus of elasticity (E) = 200 GPa = 200×10^3 N/mm²

We know that the distance between neutral axis of the wire and its extreme fibre,



$$y = \frac{d}{2} = \frac{5}{2} = 2.5 \text{ mm}$$

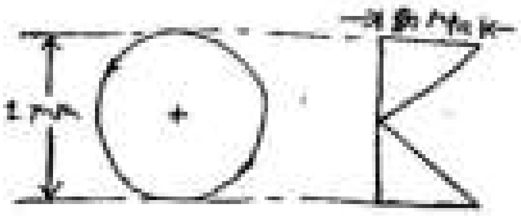
and maximum bending stress induced in the wire,

$$\sigma_{b(max)} = \frac{E}{R} \times y = \frac{200 \times 10^3}{5 \times 10^3} \times 2.5 = 100 \text{ N/mm}^2$$
$$= 100 \text{ MPa} \quad \underline{\text{Ans:}}$$

(iii) Prob (4) A copper wire of 2 mm dia is required to be wound around a drum. Find the minimum radius of the drum, if the stress in the wire is not to exceed 80 MPa. Take the modulus of elasticity for the copper as 100 GPa.

Solution: Given: Diameter of wire (d) = 2 mm, maximum bending stress $\sigma_{b(max)}$ = 80 MPa = 80 N/mm² and modulus of elasticity (E) = 100 GPa = 100×10^3 N/mm².

We know that distance between the neutral axis of the wire and its extreme fibre,



$$y = \frac{d}{2} = \frac{2}{2} = 1 \text{ mm}$$

∴ Minimum radius of the drum,

$$R = \frac{y}{\sigma_{b(max)}} \times E = \frac{1}{80} \times 100 \times 10^3$$
$$= 1.25 \times 10^3 \text{ mm} = 1.25 \text{ m} \quad \underline{\text{Ans:}}$$

P.T.O → (last)

STRENGTH OF MATERIAL (3RD MECH, DEPT)

(8) Thermal stress in composite bar :

Whenever there is some increase or decrease in the temp of a bar, consisting of two or more different materials, it causes the bar to expand or contract. On account of different coefficients of linear expansions the two materials do not expand or contract by the same amount.

(a) (b) (c)

Now consider a composite bar consisting of two members, a bar of steel and another of brass as shown in figure.

Now let, σ_1 = stress in brass,

ε_1 = strain in brass,

α_1 = coefficient of linear expansion of brass,

A_1 = cross-sectional area of brass bar,

$\sigma_2, \varepsilon_2, \alpha_2, A_2$ = corresponding values of steel, and

ε = Actual strain of the composite bar per unit length.

As the compressive load on the brass is equal to the tensile load on the steel, therefore,

$$\sigma_1 \cdot A_1 = \sigma_2 A_2 \quad [\because \sigma = \frac{P}{A}]$$

Now strain in brass, $\varepsilon_1 = \alpha_1 t - \varepsilon$ ---- (i)

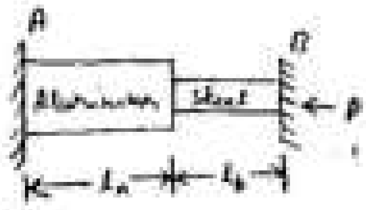
and strain in steel $\varepsilon_2 = \alpha_2 \cdot t - \varepsilon$ ---- (ii)

Adding equation (i) and (ii), we get,

$\varepsilon_1 + \varepsilon_2 =$

ⓒ Composite section under thermal stress :

Consider a composite bar rigidly fixed at supports A and B. The support reactions can be found.



Total expansion, $\Delta l = \alpha_a t\, l_a + \alpha_s t\, l_s$

$$\text{or,} \quad \Delta l = (\alpha_a l_a + \alpha_s l_s)\, t \quad \text{---- (i)}$$

Let P be the support reaction. Then

$$\Delta l = \frac{P l_a}{A_a E_a} + \frac{P l_s}{A_s E_s} \quad \text{---- (ii)}$$

From equation (i) and (ii)

$$(\alpha_a l_a + \alpha_s l_s)\, t = P\left(\frac{l_a}{A_a E_a} + \frac{l_s}{A_s E_s}\right)$$

⑨ Strain energy :

The energy which is absorbed in a body, when strain within its elastic limit, is known as strain energy. It has been experimentally found that this strain energy is always capable of doing some work.

Strain energy = work done.

⑩ Resilience :

It is used for the total strain energy stored in a body. Sometimes the resilience is also defined as the capacity of a strained body for doing work (when it spring back) on the removal of the straining force.

⑪ Strain energy stored in a body, when the load is gradually applied :

When the loading starts from zero and increases gradually till the body is fully loaded, when we lower a body with the help of a crane, the body first touches the platform on which it is to be placed,